□13□ □□□□□□:□□□

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□□□□□□□9□□□

1□□2021□•□□□□□□□□□□□□ $f(x)=xlnx$ □

□1□□□□ $f(x)$ □□□□□□□□□□

□2□□ $b>0$ □□□□□ $b^b \geq (\frac{1}{e})^{\frac{1}{e}}$ □□□ e □□□□□□□□□□

□3□□ $a>0$ □ $b>0$ □□□ $f(x)+(a+b)ln2 \geq f(a+b)-f$ □b□□

□□□□□□1□ $f'(x)=1+lnx \ (x>0)$ - - - - - □1□□

□ $f'(x) \geq 0$ □□ $lnx \geq -1 = lne^{-1}$ □

□ $e>1$ □ $\therefore x \geq \frac{1}{e}$ □

□ $f'(x)<0$ □□ $0<x<\frac{1}{e}$ □- - - - - □2□□

$\therefore f(x)$ □ $[\frac{1}{e}$ □ $+\infty)$ □□□□□□□ $(0$ □ $\frac{1}{e}]$ □□□□□□- - - - □4□□

□2□□□1□□□□ $b>0$ □□□ f □b□ $\geq f(x)_{min}=f(\frac{1}{e})=-\frac{1}{e}$ □- - - - □6□□

$\therefore blnb \geq -\frac{1}{e}$ □□□ $lnb^b \geq ln(\frac{1}{e})^{\frac{1}{e}}$ □ $\therefore b^b \geq (\frac{1}{e})^{\frac{1}{e}}$ □- - - - - □8□□

□3□□ f □a□ $+(a+b)ln2 \geq f(a+b)-f$ □b□□□□□

f □a□ $+f$ □b□ $\geq f(a+b)-(a+b)ln2$ - - - - - □7□□

□□□□ f □a□ $+f(a+b-a) \geq f(a+b)-(a+b)ln2$

□□□ $g(x)=f(x)+f(k-x)(k>0)$ - - - - - - □8□□

$\therefore g(x) = x\ln x + (k-x)\ln(k-x)$□$\therefore 0 < x < k$

$\therefore g'(x) = \ln x + 1 - \ln(k-x) - 1 = \ln\dfrac{x}{k-x}$

□$g'(x) > 0$□□□$\dfrac{x}{k-x} > 1 \Rightarrow \dfrac{2x-k}{k-x} > 0 \Rightarrow \dfrac{k}{2} < x < k$□

$\therefore g(x)$□$[\dfrac{k}{2}$□$k)$□□□□□□□$(0$□$\dfrac{k}{2}]$□□□□□□

$\therefore g(x)$□□□□□□$g(\dfrac{k}{2})$□□□□□$g(x) \geqslant g(\dfrac{k}{2})$□- - - - □12 □□

$g(\dfrac{k}{2}) = f(\dfrac{k}{2}) + \ (k-\dfrac{k}{2}) = k\ln\dfrac{k}{2} = k(\ln k - 2) = f(k) - k\ln 2$

$\therefore g(x) \geqslant f(k) - k\ln 2$□□□$f(x) + f(k-x) \geqslant f(k) - k\ln 2$□- - - - - □13 □□

□$x=a$□$k-x=b$□□$k=a+b$

$\therefore f$□a□$+ f$□b□$\geqslant f(a+b) - (a+b)\ln 2$□

$\therefore f$□a□$+(a+b)\ln 2 \geqslant f(a+b) - f$□b□□□□- - - - - - □14 □□

2□□2021 □•□□□□□□□□□$f(x) = a\dfrac{e^x}{x} + (\ln x - x)$□□□$a \in R$□$a$□□□□$e$□□□□□□□□□$e = 2.71828\cdots)$□

□I□□□□$f(x)$□□□□□□□□□□□□a□□□□□□

□II□□$a=0$□□□$f(x) \leqslant kx+m$□□□$m>0)$□□□□□$(k+1)m$□□□□$h(m)$□□□□□

□□□□□□I□□□$f(x)$□□□□□$(0,+\infty)$□

□□□□$f'(x) = a\cdot\dfrac{e^x(x-1)}{x^2} - \dfrac{x-1}{x} = \dfrac{e^x(x-1)}{x^2}(a - \dfrac{x}{e^x})$□

□$f'(x) = 0 \Rightarrow x=1$□$a = \dfrac{x}{e^x}$□

□$u(x) = \dfrac{x}{e^x}$□$u'(x) = \dfrac{1-x}{e^x}$□

∴□ $x\in(0,1)$ □□ $u(x)>0$ □□ $x\in(1,+\infty)$ □□ $u(x)<0$ □

□ $u(x)$ □□□ $(0,1)$ □□□□□□ $(1,+\infty)$ □□□□

∴ $u(x)_{□□}=u(1)=\frac{1}{e}$ □

□□ $x\to 0$ □□ $u(x)\to 0$ □□ $x\to+\infty$ □□ $u(x)\to 0$ □ $u(x)>0$ □□□□

∴□ $a\leqslant 0$ □ $a>\frac{1}{e}$ □□□□ $a=\frac{x}{e^x}$ □□□□□ $f(x)$ □□ $x=1$ □□□□□□

□ $a=\frac{1}{e}$ □□□□ $a=\frac{x}{e^x}$ □□□□ $x=1$ □□□ $f'(x)$ □□□ $a-\frac{x}{e^x}\geqslant 0$ □□□□

□□□ $f(x)$ □□ $x=1$ □□□□□□

□ $0<a<\frac{1}{e}$ □□□□ $a=\frac{x}{e^x}$ □□□□ x_1 □ x_2 □ $x_1\in(0,1)$ □ $x_2\in(1,+\infty)$ □

∴□□ $f(x)$ □□□ $(0,x_1)$ □□□□□ $(x_1$ □ $1)$ □□□□□ $(1,x_2)$ □□□□□ $(x_2$ □ $+\infty)$ □□□□□□□□□ $f(x)$ □ x_1 □1□ x_2 □□□□□□

□□□□□□ $a\leqslant 0$ □ $a\geqslant\frac{1}{e}$ □□□□ $f(x)$ □□□□□□□□

□Ⅱ□□□□□ $\ln x-x\leqslant kx+m$ □□ $\varphi(x)=\ln x-(k+1)x-m$ □□□ $\forall x\in(0,+\infty)$ □□□ $\varphi(x)\leqslant 0$ □□□

□ $\varphi'(x)=\frac{1}{x}-(k+1)$ □∴□ $k+1\leqslant 0$ □□□□ $\varphi(x)$ □ $(0,+\infty)$ □□□□□□

□□□ $\varphi(e^m)=-(k+1)e^m\geqslant 0$ □

∴□ $x\in(e^m$ □ $+\infty)$ □□ $\varphi(x)>0$ □□□□□ $\varphi(x)\leqslant 0$ □□□□□□

□ $k+1>0$ □□□□ $\varphi'(x)$ □ $(0,+\infty)$ □□□□□□□ $\varphi'(\frac{1}{k+1})=0$ □

∴□□ $\varphi(x)$ □□□ $(0,\frac{1}{k+1})$ □□□□□□□ $(\frac{1}{k+1},+\infty)$ □□□□□□

∴ $\varphi(x)\leqslant\varphi(\frac{1}{k+1})=-\ln(k+1)-1-m$ □

□□ $\forall x\in(0,+\infty)$ □□□ $\varphi(x)\leqslant 0$ □□□□□□ $-\ln(k+1)-1-m\leqslant 0$ □□□

$\therefore ln(k+1) \geq -1-m \Rightarrow k+1 \geq e^{-1-m}$□

□ $m>0$□□□ $(k+1)m$ □□□□ $h(m)=me^{-1-m}$□

$h'(m)=e^{-1-m}(1-m)$□

$\therefore$□□ $h(m)$ □ $(0,1)$ □□□□□ $(1,+\infty)$ □□□□

$\therefore h(m) \leq \frac{1}{e^2}$ □□ $(k+1)m$ □□□□ $h(m)$ □□□□□ $\frac{1}{e^2}$ □

□□□□□ $(k+1)m$ □□□□ $h(m)$ □□□□□ $\frac{1}{e^2}$ □

3□□2021•□□□□□□□□□□□ $f(x)=xlnx$□

□I□ □ $f(x)$ □□□□

□II□□ $g(x)=f(x+1)$ □□□□□□ $x \geq 0$ □□□ $g(x) \geq mx$ □□□□□□ m □□□□□□

□III□□ $0<a<b$ □□□□ $0<f(a)+f(b)-2f(\frac{a+b}{2})<(b-a)ln2$ □

□□□□□□□□□14 □□

□□□I□□□ $f(x)=xlnx$ □□ $f'(x)=1+lnx$ □ $(x>0)$

□ $f'(x)=0$ □□□□ $x=\frac{1}{e}$ □□□ $x\in(0,\frac{1}{e})$ □□ $f'(x)<0$ □ $x\in(\frac{1}{e},+\infty)$ □□ $f'(x)>0$

□□□ $f(x)$ □□□□□ $f(\frac{1}{e})=-\frac{1}{e}$

□II□ $g(x)=f(x+1)=(x+1)ln(x+1)$

□ $h(x)=(x+1)ln(x+1)-mx$ □□ $h'(x)=ln(x+1)+1-m$

□□□□ $h(0)=0$ □□□ $h(x) \geq 0$ □□□□□ $h'(0) \geq 0$ □□ $1-m \geq 0$ □□□ $m \leq 1$

□□□□□□ $m \leq 1$ □□ $h'(x)=ln(x+1)+1-m \geq 0$ □□□□

□□□m□□□□□□□$m\leqslant 1$

(III)□□□□$F(x)=a\ln a+x\ln x-(a+x)\ln\frac{a+x}{2}$□$x>a$□$F'(x)=1+\ln x-\ln\frac{a+x}{2}-1=\ln\frac{2x}{a+x}$□

□ $x>a$□$\therefore 0<a+x<2x$□$F'(x)>0$□$F(x)$□$(a,+\infty)$□□□□□□□□

□F□b□$>F$□a□$=0$□□□$f(a)+f(b)-2f(\frac{a+b}{2})>0$

□□□□□□□□□$G(x)=a\ln a+x\ln x-(a+x)\ln\frac{a+x}{2}-(x-a)\ln 2$□

$G'(x)=\ln\frac{2x}{a+x}-\ln 2=\ln\frac{x}{a+x}<0$□

$G(x)$□$(a,+\infty)$□□□□□□□

□G□b□$<G$□a□$=0$□□$f(a)+f(b)-2f(\frac{a+b}{2})<(b-a)\ln 2$

□□□$0<f(a)+f(b)-2f(\frac{a+b}{2})<(b-a)\ln 2$□

4□□2021•□□□□□□□□□$f(x)=e^x-x$□$g(x)=(x+k)\ln(x+k)-x$□

□1□□$k=1$□$f'(t)=g'(t)$□□□□t□□□

□2□□a□$b\in R$□f□a□$+g$□b□$\geqslant f(0)+g(0)+ab$□□□□□k□□□□□□

□□□□□□1□□ □□$f(x)=e^x-x$□$g(x)=(x+k)\ln(x+k)-x$□

$\therefore f'(x)=e^x-1$□$g'(x)=\ln(x+k)$□

□$k=1$□$f'(t)=g'(t)$□□$e^t-\ln(t+1)-1=0$□

□$\varphi(t)=e^t-\ln(t+1)-1$□□$\varphi'(t)=e^t-\frac{1}{t+1}$□

□ $\varphi''(t)=e^t+\frac{1}{(t+1)^2}>0$□$\therefore\varphi'(t)$□$(-1,+\infty)$□□□□□

□$\varphi'(0)=0$□∴□$-1<x<0$□□$\varphi'(t)>0$□$\varphi(t)$□□□□□

□$x>0$□□$\varphi'(t)<0$□$\varphi(t)$□□□□□

$\therefore \varphi(t)\leqslant \varphi(0)=0$□

□□□□$t=0$□□□□□□

∴□□$f'(t)=g'(t)$□□□□□□□$t=0$□□□t□□□0□

□2□□$h(x)=f(x)-bx+g$□b□$-f(0)-g(0)$□$x>0$□

□$h'(x)=e^{x}-(b+1)$□

∴□$x>\ln(b+1)$□□$h'(x)>0$□$h(x)$□□□□□

□$0<x<\ln(b+1)$□□$h'(x)<0$□$h(x)$□□□□□

□$h(x)\geqslant h(\ln(b+1))=f(\ln(b+1))+g$□b□$-f(0)-g(0)-b\ln(b+1)$

$=(b+k)\ln(b+k)-(x+1)\ln(x+1)-k\ln k$□$(x>0)$□

□$t(x)=(x+k)\ln(x+k)-(x+1)\ln(x+1)-k\ln k$□$(x>0)$□

□$t'(x)=\ln(x+k)-\ln(x+1)$□

(i)□$k>1$□□$t'(x)>0$□$t(x)$□$(0,+\infty)$□□□□□

$\therefore t(x)>t(0)=0$□□□□□□

(ii)□$k=1$□□$t(x)=0$□□□□□□

(iii)□$0<k<1$□□$t'(x)<0$□$t(x)$□$(0,+\infty)$□□□□□

$\therefore t(x)<t(0)=0$□□□□□□□

□□□□□ k □□□□□ $[1$ □ $+\infty)$ □

5□□2021•□□□□□□ $a\neq 0$ □□□ $f(x)=a\ln x+\sqrt{1+x}$ □ $x>0$ □

□I□□ $a=-\frac{3}{4}$ □□□□ $f(x)$ □□□□□

□II□□□□ $x\in[\frac{1}{e^2}$ □ $+\infty)$ □□ $f(x)\leq\frac{\sqrt{x}}{2a}$ □□ a □□□□□□

□□ $e=2.71828\cdots$ □□□□□□□□

□□□□□□1□□ $a=-\frac{3}{4}$ □□ $f(x)=-\frac{3}{4}\ln x+\sqrt{1+x}$ □ $x>0$ □

$f'(x)=-\frac{3}{4x}+\frac{1}{2\sqrt{1+x}}=\frac{(\sqrt{1+x}-2)(2\sqrt{1+x}+1)}{4x\sqrt{1+x}}$ □

$\therefore$ □□ $f(x)$ □□□□□□□□ $(0,3)$ □□□□□□□□ $(3,+\infty)$ □

□2□□ f □1□ $\leq\frac{1}{2a}$ □□ $0<a\leq\frac{\sqrt{2}}{4}$ □

□ $0<a\leq\frac{\sqrt{2}}{4}$ □□ $f(x)\leq\frac{\sqrt{x}}{2a}$ □□□□ $\frac{\sqrt{x}}{a^2}-\frac{2\sqrt{1+x}}{a}-2\ln x\geq 0$ □

□ $t=\frac{1}{a}$ □□ $t\geq 2\sqrt{2}$ □

□ $g(t)=t^2\sqrt{x}-2t\sqrt{1+x}-2\ln x$ □ $t\geq 2\sqrt{2}$ □

□ $g(t)=\sqrt{x}(t-\sqrt{1+\frac{1}{x}})^2-\frac{1+x}{\sqrt{x}}-2\ln x$ □

(i) □ $x\in[\frac{1}{7}$ □ $+\infty)$ □□ $\sqrt{1+\frac{1}{x}}\leq 2\sqrt{2}$ □

□ $g(x)\geq g(2\sqrt{2})=8\sqrt{x}-4\sqrt{2}\sqrt{1+x}-2\ln x$ □

□ $p(x)=4\sqrt{x}-2\sqrt{2}\sqrt{1+x}-\ln x$ □ $x\geq\frac{1}{7}$ □

□ $p'(x)=\frac{2}{\sqrt{x}}-\frac{\sqrt{2}}{\sqrt{x+1}}-\frac{1}{x}=\frac{2\sqrt{x}\sqrt{x+1}-\sqrt{2}x-\sqrt{x+1}}{x\sqrt{x+1}}$

$=\frac{(x-1)[1+\sqrt{x}(\sqrt{2x+2}-1)]}{x\sqrt{x+1}(\sqrt{x}+1)(\sqrt{x+1}+\sqrt{2x})}$ □

□□□□□

x	$\frac{1}{7}$	$(\frac{1}{7}$ □ $1)$	1	$(1,+\infty)$
$p'(x)$		-	0	+
$P(x)$	$p(\frac{1}{7})$	□□□□	□□□ p □1□	□□□□

$\therefore p(x)\geq p$ □1□ $=0$ □

$\therefore g(t)\geq g(2\sqrt{2})=2p(x)=2p(x)\geq 0$ □

(ii) □ $x\in[\frac{1}{e^2},\frac{1}{7})$ □□ $g(t)\geq g(\sqrt{1+\frac{1}{x}})=\frac{-2\sqrt{x}\ln x-(x+1)}{2\sqrt{x}}$ □

□ $q(x)=2\sqrt{x}\ln x+(x+1)$ □ $x\in[\frac{1}{e^2}$ □ $\frac{1}{7}]$ □

□ $q'(x)=\frac{lnx+2}{\sqrt{x}}+1>0$ □

□ $q(x)$ □ $[\frac{1}{e^{2}}$ □ $\frac{1}{7}]$ □□□□□□ $\therefore q(x)\leqslant q(\frac{1}{7})$ □

□ (i) □ $q(\frac{1}{7})=-\frac{2\sqrt{7}}{7}p(\frac{1}{7})<-\frac{2\sqrt{7}}{7}p$ □1□ $=0$ □

$\therefore q(x)<0$ □ $\therefore g(t)\geqslant g(\sqrt{1+\frac{1}{x}})=-\frac{q(x)}{2\sqrt{x}}>0$ □

□ $(i)(ii)$ □□□ $x\in[\frac{1}{e^{2}}$ □ $+\infty)$ □ $t\in[2\sqrt{2}$ □ $+\infty)$ □ $g(t)\geqslant 0$ □

□□□□ $x\in[\frac{1}{e^{2}}$ □ $+\infty)$ □□□ $f(x)\leqslant\frac{\sqrt{x}}{2a}$ □

□□□□□□□□ a □□□□□□ $(0$ □ $\frac{\sqrt{2}}{4}]$ □

6□□2021•□□□□□ $f(x)=(x-a)(x-b)(x-c)$ □ a □ b □ $c\in R$ □ $f'(x)$ □ $f(x)$ □□□□□

□1□□ $a=b=c$ □ f □4□ $=8$ □□ a □□□

□2□□ $a\neq b$ □ $b=c$ □□ $f(x)$ □ $f'(x)$ □□□□□□□ $\{-3$ □1□ $3\}$ □□□ $f(x)$ □□□□□

□3□□ $a=0$ □ $0<b\leqslant 1$ □ $c=1$ □□ $f(x)$ □□□□ M □□□□ $M\leqslant\frac{4}{27}$ □

□□□□□□1□ $\because a=b=c$ □ $\therefore f(x)=(x-a)^{3}$ □

$\because f$ □4□ $=8$ □ $\therefore(4-a)^{3}=8$ □

$\therefore 4-a=2$□□□$a=2$□

□2□$a\neq b$□$b=c$□□$f(x)=(x-a)(x-b)^2$□

□$f(x)=(x-a)(x-b)^2=0$□□□$x=a$□□$x=b$□

$f'(x)=(x-b)^2+2(x-a)(x-b)=(x-b)(3x-b-2a)$□

□$f'(x)=0$□□□$x=b$□□$x=\dfrac{2a+b}{3}$□

$\because$ $f(x)$□$f'(x)$□□□□□□□$A=\{-3$□1□$3\}$□□

□□$a=-3$□$b=1$□□$\dfrac{2a+b}{3}=\dfrac{-6+1}{3}=-\dfrac{5}{3}\notin A$□□□□

$a=1$□$b=-3$□□$\dfrac{2a+b}{3}=\dfrac{2-3}{3}=-\dfrac{1}{3}\notin A$□□□□

$a=-3$□$b=3$□□$\dfrac{2a+b}{3}=\dfrac{-6+3}{3}=-1\notin A$□□□□□

$a=3$□$b=1$□□$\dfrac{2a+b}{3}=\dfrac{6+1}{3}=\dfrac{7}{3}\notin A$□□□□

$a=1$□$b=3$□□$\dfrac{2a+b}{3}=\dfrac{5}{3}\notin A$□□□□

$a=3$□$b=-3$□□$\dfrac{2a+b}{3}=\dfrac{6-3}{3}=1\in A$□

□□$a=3$□$b=-3$□$\dfrac{2a+b}{3}=1\in A$□

□□□$f(x)=(x-3)(x+3)^2$□

$f'(x)=3[x-(-3)](x-1)$□

□□$x=1$□□□□$f(x)$□□□□□□f□1□$=-2\times 4^2=-32$□

□3□□□□$a=0$□$0<b\leq 1$□$c=1$□

$f(x)=x(x-b)(x-1)$□

$f'(x)=(x-b)(x-1)+x(x-1)+x(x-b)=3x^2-(2b+2)x+b$□

△ $=4(b+1)^2-12b=4b^2-4b+4=4(b-\frac{1}{2})^2+3..3$ □

□ $f'(x)=3x^2-(2b+2)x+b=0$□

□□□ $x_1=\frac{b+1-\sqrt{b^2-b+1}}{3}\in(0,\frac{1}{3}]$□ $x_2=\frac{b+1+\sqrt{b^2-b+1}}{3}$□ $x_1<x_2$□

$x_1+x_2=\frac{2b+2}{3}$□ $x_1x_2=\frac{b}{3}$□

□□ $x=x_1$ □□ $f(x)$ □□□□□ M□

∵ $f'(x_1)=3x_1^2-(2b+2)x_1+b=0$□□ $x_1=t\in(0,\frac{1}{3}]$□

□□□ $b=\frac{3t^2-2t}{2t-1}$□

$\therefore M=f(x_1)=x_1(x_1-b)(x_1-1)=t(t-b)(t-1)=\frac{-t^4+2t^3-t^2}{2t-1}$□

$M=\frac{-6t^4+12t^3-8t^2+2t}{(2t-1)^2}$□

□ $g(t)=-6t^3+12t^2-8t+2$□

$g(t)=-18t^2+24t-8=-2(3t-2)^2<0$□

$\therefore$□□ $g(t)$□ $t\in(0,\frac{1}{3}]$ □□□□□ $g(\frac{1}{3})=\frac{4}{9}>0$□

$\therefore t\cdot g(t)>0$□$\therefore M>0$□

$\therefore$□□ $M(t)$□ $t\in(0,\frac{1}{3}]$ □□□□□

$\therefore M(t)_{,,}\ M(\frac{1}{3})=\frac{4}{27}$□

7□□2021□•□□□□□□□□□ $f(x)=1-\frac{1}{x}+a\ln x$ □ $(a\in R)$ □

□1□□□□ $f(x)$ □□□□□

□2□□□ $g(x)=2(x+1)+xf(x)$ □□□□□ $0<a\leqslant 1$ □□ $g(x)>0$ □□□□

□□□□□□1□ $f'(x)=\frac{1}{x^2}+\frac{a}{x}=\frac{1+ax}{x^2}$ □□1 □□

□ $a\geqslant 0$ □□ $f'(x)>0$ □ $\therefore f(x)$ □□□□□□□ $(0,+\infty)$ □□2 □□

□ $a<0$ □□□ $f'(x)>0\Rightarrow 0<x<-\frac{1}{a}$ □□ $f'(x)<0\Rightarrow x>-\frac{1}{a}$ □□3 □□

$\therefore f(x)$ □□□□□□□ $(0,-\frac{1}{a})$ □□□□□□□ $(-\frac{1}{a},+\infty)$ □□4 □□

□2□ $g(x)=2(x+1)+x-1+ax\ln x=3x+1+ax\ln x(x>0)$ □

□□□□□□□ $g'(x)=3+a(\ln x+1)=a\ln x+3+a$ □□ $g'(x)=0\Rightarrow x=e^{\frac{-(3+a)}{a}}$ □□5 □□

□ $a>0$ □□ $g'(x)>0\Rightarrow x>e^{\frac{-(3+a)}{a}}$ □□ $g'(x)<0\Rightarrow 0<x<e^{\frac{-(3+a)}{a}}$ □

$\therefore g(x)$ □ $\left(0,e^{\frac{-(3+a)}{a}}\right)\downarrow,\left(e^{\frac{-(3+a)}{a}},+\infty\right)\uparrow$ □□6 □□

$\therefore g(x)\geqslant g(e^{\frac{-(3+a)}{a}})$ □ $g(e^{\frac{-(3+a)}{a}})=3\cdot e^{\frac{-(3+a)}{a}}+1+a\cdot e^{\frac{-(3+a)}{a}}\cdot\frac{-(3+a)}{a}$ □□7 □□

□ $\frac{-(3+a)}{a}=t(t\leqslant -4)$，□ $g(t)=3\cdot e^t+1+\frac{-3t}{t+1}\cdot e^t(t\leqslant -4)$ □□8 □□

□□□□ $3\cdot e^t+1+\frac{-3t}{t+1}\cdot e^t>0$ □

□□ $1+\frac{3e^t}{t+1}>0$ □□ $h(t)=1+\frac{3e^t}{t+1},(t\leqslant -4)$ □□9 □□

□ $h'(t)=\frac{3te^t}{(t+1)^2}<0$ □ $\therefore h(t)$ □ $(-\infty,-4)$ □□□

∴ $h(t) \geq h(-4) = 1-\frac{1}{e^{4}} > 0$ □∴ $1+\frac{3e^{t}}{t+1} > 0$ □□11 □□

∴□ $0 < a \leq 1$ □□ $g(x) > 0$ □□□□□12 □□

□□□□ $g(x) = 3x+1+ax\ln x = x(3+\frac{1}{x}+a\ln x),(x>0)$ □□□ $g(x) > 0$ □□□□ $3+\frac{1}{x}+a\ln x > 0$ □□5 □□

□ $h(x) = 3+\frac{1}{x}+a\ln x$，□ $h'(x) = -\frac{1}{x^2}+\frac{a}{x} = \frac{ax-1}{x^2}$ □□6 □□

□ $h'(x) > 0 \Rightarrow x > \frac{1}{a}, h'(x) < 0 \Rightarrow 0 < x < \frac{1}{a}$ □□7 □□

$h(x)$ □ $\left(0,\frac{1}{a}\right)\downarrow,\left(\frac{1}{a},+\infty\right)\uparrow$ □∴ $h(x) \geq h(\frac{1}{a}), h(\frac{1}{a}) = 3+a-a\ln a$ □□8 □□

□□□□ 1□ $h(x)_{min} = h(\frac{1}{a}) = 3+a-a\ln a$ □□ $a\in(0$ □ $1]$ □∴ $\ln a \leq 0$ □□9 □□

∴ $-a\ln a \geq 0$ □□10 □□□

∴ $3+a-a\ln a > 0$ □□11 □□

∴□ $0 < a \leq 1$ □□ $g(x) > 0$ □□□□□12 □□

□ □ □ □ 2□ $h(\frac{1}{a}) = 3+a-a\ln a = a(\frac{3}{a}+1-\ln a)$ □ □ □ □ □ □ $\frac{3}{a}+1-\ln a > 0$ □ □

$r(a) = \frac{3}{a}+1-\ln a,(0 < a \leq 1)$，□ $r'(a) = -\frac{3}{a^2}-\frac{1}{a} < 0$ □

∴ r□a□□ $(0,1)$ □□□10 □□

∴ r□a□ $\geq r$□1□ $=4>0$ □∴ $\frac{3}{a}+1-\ln a > 0$ □□11 □□

∴□ $0 < a \leq 1$ □□ $g(x) > 0$ □□□□□12 □□

8．□2021•□□□□□□ $f(x) = x^2+k\ln x(k\in R)$ □ $f'(x)$ □ $f(x)$ □□□□□

□I□□ $k=6$ □□

□i□□□□ $y=f(x)$ □□ $(1$ □ f' □1□ $)$ □□□□□□□

□ii□□□□ $g(x)=f(x)-f'(x)+\frac{9}{x}$ □□□□□□□□

□II□□ $k\geq -3$ □□□□□□□□□ x_1 □ $x_2\in[1$ □ $+\infty)$ □□ $x_1>x_2$ □□ $\frac{f'(x_1)+f'(x_2)}{2}>\frac{f(x_1)-f(x_2)}{x_1-x_2}$ □

□□□□□□ $(I)(i)$ □ $k=6$ □□ $f(x)=x^3+6\ln x$ □

□ $f'(x)=3x^2+\frac{6}{x}$ □

$\therefore f'$ □1□ $=9$ □

□ f □1□ $=1$ □

$\therefore$ □□ $y=f(x)$ □□ $(1$ □ f □1□ $)$ □□□□□□□ $y-1=9(x-1)$ □□ $9x-y-8=0$ □

$(ii)\ g(x)=f(x)-f'(x)+\frac{9}{x}=x^3+6\ln x-3x^2+\frac{3}{x}$ □ $x>0$ □

$\therefore g'(x)=3x^2-6x+\frac{6}{x}-\frac{3}{x^2}=\frac{3(x-1)^3(x+1)}{x^2}$ □

□ $g'(x)=0$ □□□ $x=1$ □

□ $0<x<1$ □ $g'(x)<0$ □

□ $x>1$ □ $g'(x)>0$ □

$\therefore$ □□ $g(x)$ □ $(0,1)$ □□□□□□□ $(1,+\infty)$ □□□□□□

$x=1$ □□□□□□□□□□ g □1□ $=1$ □□□□□

□II□□□□□ $f(x)=x^3+k\ln x$ □□ $f'(x)=3x^2+\frac{k}{x}$ □

□□□□ x_1 □ $x_2\in[1$ □ $+\infty)$ □□ $x_1>x_2$ □□ $\frac{x_1}{x_2}=t$ □ $t>1$ □

□ $(x_1-x_2)[f'(x_1)+f'(x_2)]-2[f(x_1)-f(x_2)]=(x_1-x_2)(3x_1^2+\frac{k}{x_1}+3x_2^2+\frac{k}{x_2})-2(x_1^3-x_2^3+kln\frac{x_1}{x_2})$ □

$=x_1^3-x_2^3-3x_1^2x_2+3x_1x_2^2+k(\frac{x_1}{x_2}-\frac{x_2}{x_1})-2kln\frac{x_1}{x_2}$ □

$=x_2^3(t^3-3t^2+3t-1)+k(t-\frac{1}{t}-2lnt)$ □①

□ $h(x)=x-\frac{1}{x}-2lnx$ □ $x>1$ □

□ $x>1$ □□ $h'(x)=1+\frac{1}{x^2}-\frac{2}{x}=(1-\frac{1}{x})^2>0$ □

$\therefore h(x)$ □ $(1,+\infty)$ □□□□□

$\therefore$ □ $t>1$ □ $h(t)>h$ □1□ $=0$ □□ $t-\frac{1}{t}-2lnt>0$ □

□ $x_2\geq 1$ □ $t^3-3t^2+3t-1=(t-1)^3>0$ □ $k\geq -3$ □

$\therefore x_2^3(t^3-3t^2+3t-1)+k(t-\frac{1}{t}-2lnt)\geq t^3-3t^2+3t-1-3(t-\frac{1}{t}-2lnt)=t^3-3t^2+6lnt+\frac{3}{t}-1$ □②□

□□I□ (ii) □□□ $t\geq 1$ □□ $g(t)>g$ □1□

□ $t^3-3t^2+6lnt+\frac{3}{t}>1$ □③□

□①②③□□ $(x_1-x_2)[f'(x_1)+f'(x_2)]-2[f(x_1)-f(x_2)]>0$ □

$\therefore$ □ $k\geq -3$ □□□□□□ x_1 □ $x_2\in[1$ □ $+\infty)$ □□ $x_1>x_2$ □□ $\frac{f'(x_1)+f'(x_2)}{2}>\frac{f(x_1)-f(x_2)}{x_1-x_2}$ □

9□□2021•□□□□□□□□ $f(x)=lnx-x+1$ □

□1□□□ $f(x)$ □□□□□

□2□□□□ $x\in(1,+\infty)$ □□ $1<\frac{x-1}{lnx}<x$ □

□3□□ $c>1$□□□□ $x\in(0,1)$ □□ $1+(c-1)x>c^{x}$ □

□□□□□□1□□□ $f(x)=lnx-x+1$□□□□ $f'(x)=\frac{1}{x}-1$ □ $x>0$□

□ $f'(x)>0$□□□ $0<x<1$□□ $f'(x)<0$□□□ $x>1$□

□□ $f(x)$ □□□□□ $(0,1)$ □□□□□ $(1,+\infty)$ □

□2□□□□□ $x\in(1,+\infty)$ □□ $1<\frac{x-1}{lnx}<x$ □□□ $lnx<x-1<xlnx$□

□□1□□□ $f(x)=lnx-x+1$ □ $(1,+\infty)$ □□□

□□ $f(x)<f$□1□$=0$□□□ $lnx<x-1$□

□ $F(x)=xlnx-x+1$□ $x>1$□ $F'(x)=1+lnx-1=lnx$□

□ $x>1$□□ $F'(x)>0$□□□ $F(x)$ □□□□□ $F(x)>F$□1□$=0$□

□□ $xlnx>x-1$□□□□□□□□□

□3□□□□□ $G(x)=1+(c-1)x-c^{x}$□

□□□□□□□ $x\in(0,1)$ □□ $G(x)>0(c>1)$□

$G'(x)=c-1-c^{x}lnc$□ $G''(x)=-(lnc)^{2}c^{x}<0$□

$\therefore G'(x)$ □ $(0,1)$ □□□□□□ $G'(0)=c-1-lnc$□ G'□1□$=c-1-clnc$□

□□1□□ $f(x)$ □□□□□□□ $G'(0)=c-1-lnc>0$□□□2□□□ G'□1□$=c-1-clnc=c(1-lnc)-1<0$□

$\therefore \exists t\in(0,1)$□□□ $G'(t)=0$□□ $x\in(0,t)$ □□ $G'(x)>0$□ $x\in(t,1)$ □□ $G'(x)<0$□

□ $G(x)$ □ $(0,t)$ □□□□ $(t,1)$ □□□

□□□□ $G(0)=G$□1□$=0$□

$\therefore x \in (0,1)$ [illegible] $G(x) > 0$ [illegible]

[illegible] $c > 1$ [illegible] $x \in (0,1)$ [illegible] $1 + (c - 1)x > c^x$ [illegible]